On the Quintessence with Abelian and Non-abelian Symmetry

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Abstract

We study the perturbations on both "radial" and "angular" components of the quintessence with an internal abelian and non-abelian symmetry. The properties of the perturbation on the "radial" component depend on the specific potential of the model and is similiar for both abelian and non-abelian case. We show that the consine-type potential is very interesting for the O(N) quintessence model and also give a critical condition of instability for the potential. While the properties of perturbations on "angular" components depend on whether the internal symmetry is abelian or non-abelian, which we have discussed respectively. In the non-abelian case, the fluctuation of the "angular" component will increase rapidly with time while in the abelian case it will not.

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1. Introduction

Recent observations[1, 2, 3] show that our Universe is flat and 73 percent of its total energy density is resulted from "dark energy", which has a negative pressure and can accelerate the expansion of the Universe[4, 5]. using recent announcement by the WMAP satellite[3], we still do not judge whether dark energy is due to an unchanging, uniform and inert "vacuum energy" (also known as a cosmological constant) or a dynamic cosmic field that changes with time and varies across space (known as quintessence). Quintessence can be considered as a spatially homogeneous scalar field that evolves with time and many models[6, 7, 8, 9, 10, 11] have been constructed so far. The possibility of quintessence as rolling tachyon has also been investigated [12].

Boyle et al[13] and Gu and Hwang[14] have discussed quintessence with complex scalar field. In a previous paper [15], we have further generalized their ideas by replacing the complex scalar field with a N-plet scalar field which is spinning in a O(N)-symmetric potential. When one of the angular components is fixed, this O(N) quintessence model will reduce to the O(N-1) quintessence model. If the N-2 angular components are fixed, O(N) quintessence model can reduce to the complex scalar model mentioned above. If all angular components are fixed, O(N) quintessence model will reduce to the quintessence.

It is worth noting that this generalization does not hold its importance for the widely studied tracker-type potentials because the amplitude of the scalar field will increase steadily and make the angular contribution negligible. But for another type widely investigated potential, the cosine-type potential[9, 15], it will be very interesting. It has been pointed out that a natural introduction of quintessence is an ultralight axion with an almost massless quark[9] and the cosine-type potential can be derived from such a model. Furthermore, This potential requires that the amplitude of the field should not goes to be very large and therefore, unlike the tracker-type potential, make the angular contribution significant.

It would be very interesting to study the behaviours of the field when it is perturbed. We firstly investigate the fluctuation of the "radial" component and find that its stability depends on the specific potential of the model while has nothing to do with whether the the internal symmetry is abelian and non-abelian. We give a critical condition of the instability for the "radial" perturbation, under which the "radial" fluctuation will grow rapidly with time. Fortunately, we find that this condition is not satisfed by most potentials for quintessence models. While the fluctuation of the "angular" components depend heavily

on whether the internal symmetry is abelian or non-abelian. In the abelian case ($N = 2$), the "angular" fluctuation will always damp, but in the non-abelian case($N > 2$), there are possibilities for the "angular" perturbations to grow rapidly with time and become strongly space-dependent. It is worth noting that when we take the metric fluctuations(which are generally small compared with the fluctuations of the quintessence field) into account, the above conclusions are still held. In order to make the discussion more clear, in this paper we will restrict ourselves to the $N = 2$ and $N = 3$ cases which represent the abelian and nonabelian symmetry respectively.

2. The Model

We start from the flat Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \tag{0.1}$$

The Lagrangian density for the quintessence with O(3) symmetry is

$$L_\Phi = \frac{1}{2}g^{\mu\nu}(\partial_\mu \Phi^a)(\partial_\nu \Phi^a) - V(|\Phi^a|) \tag{0.2}$$

where Φ^a is the component of the scalar field, $a = 1, 2, 3$. To make it possess a O(3) symmetry, we write it in the following form

$$\begin{aligned}
\Phi^1 &= R(t)\cos\varphi_1(t) \\
\Phi^2 &= R(t)\sin\varphi_1(t)\cos\varphi_2(t) \\
\Phi^3 &= R(t)\sin\varphi_1(t)\sin\varphi_2(t)
\end{aligned} \tag{0.3}$$

Therefore, we have $|\Phi^a| = R$ and assume that the potential of the O(3) quintessence depends only on R. It is clear that when we set the component φ_2 to zero, the above O(3) system will reduce to the O(2) abelian case.

The Einstein equations and equations of motion for the scalar fields can be written as

$$H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}[\frac{1}{2}(\dot{R}^2 + \frac{\Omega^2}{a^6 R^2}) + V(R)] \tag{0.4}$$

$$(\frac{\ddot{a}}{a}) = -\frac{8\pi G}{3}[\dot{R}^2 + \frac{\Omega^2}{a^6 R^2} - V(R)] \tag{0.5}$$

$$\ddot{R} + 3H\dot{R} - \frac{\Omega^2}{a^6 R^3} + \frac{\partial V(R)}{\partial R} = 0 \tag{0.6}$$

$$\ddot{\varphi}_1 + (3H + 2\frac{\dot{R}}{R})\dot{\varphi}_1 - \sin\varphi_1 \cos\varphi_1 \dot{\varphi}_2{}^2 = 0 \tag{0.7}$$

$$\ddot{\varphi}_2 + (3H + 2\frac{\dot{R}}{R})\dot{\varphi}_2 + 2\cot\varphi_1 \dot{\varphi}_1 \dot{\varphi}_2 = 0 \tag{0.8}$$

where H is Hubble parameter and the term $\frac{\Omega^2}{a^6 R^2}$ comes from the first integrals of the equations of motion for the angular components(detailed discussion see[15]).

3. General Equations of Motion for "Radial" and "Angular" Perturbation

In order to eliminate the ambiguity from gauge freedom, one has to identify gauge invariant quantities or choose a given gauge and perform the calculations of perturbation in that gauge. In this paper, we will carry out our investigation in synchronous gauge just as Ratra and Peebles have done in Ref.[4]. As we shall see, in synchronous gauge the evolution of the modes that we consider here depend on the gravitational field only through a gauge invariant quantity $\frac{\dot{h}}{2}$, so we can consider a single perturbation mode conveniently in the following sections.

The line element of perturbed metric of a spatially flat FRW spacetime is taken as

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} - h_{ij})dx^i dx^j \tag{0.9}$$

where h_{ij} are the metric fluctuations and $\mid h_{ij} \mid << 1$. For simplicity, we write the first-order equations of perturbations for the $N = 3$ case. Decomposing the components of the field as follow

$$R(t, \mathbf{x}) = R(t) + \delta R(t, \mathbf{x}) \tag{0.10}$$

$$\varphi_1(t, \mathbf{x}) = \varphi_1(t) + \delta\varphi_1(t, \mathbf{x}) \tag{0.11}$$

$$\varphi_2(t, \mathbf{x}) = \varphi_2(t) + \delta\varphi_2(t, \mathbf{x}) \tag{0.12}$$

We can obtain the equations of motion for the fluctuations as

$$
\begin{aligned}
&(\delta \ddot{R}) - \frac{1}{a^2}\nabla^2(\delta R) - [\dot{\varphi_1}^2 + \sin^2\varphi_1](\delta R) + 3\frac{\dot{a}}{a}(\delta \dot{R}) \\
&+V^{''}(R)(\delta R) - \frac{1}{2}\dot{h}\dot{R} - 2\dot{\varphi}_1 R(\delta\dot{\varphi}_1) \\
&-R\sin(2\varphi_1)\dot{\varphi_2}^2(\delta\varphi_1) - 2R\sin^2(\varphi_1)\dot{\varphi}_2(\delta\dot{\varphi}_2) = 0
\end{aligned}
\tag{0.13}
$$

$$
\begin{aligned}
&(\delta\ddot{\varphi}_1) - \frac{1}{a^2}\nabla^2(\delta\varphi_1) + 3\frac{\dot{a}}{a}(\delta\dot{\varphi}_1) + 2\frac{\dot{R}}{R}(\delta\dot{\varphi}_1) \\
&-\dot{\varphi_2}^2\cos(2\varphi_1)(\delta\varphi_1) - \frac{1}{2}\dot{h}\dot{\varphi}_1 - \sin(2\varphi_1)\dot{\varphi}_2(\delta\dot{\varphi}_2) \\
&+\frac{2}{R}\dot{\varphi}_1(\delta\dot{R}) - \frac{2}{R^2}\dot{R}\dot{\varphi}_1(\delta R) = 0
\end{aligned}
\tag{0.14}
$$

$$
\begin{aligned}
&(\delta\ddot{\varphi}_2) - \frac{1}{a^2}\nabla^2(\delta\varphi_2) + 3\frac{\dot{a}}{a}(\delta\dot{\varphi}_2) + 2\frac{\dot{R}}{R}(\delta\dot{\varphi}_2) - \frac{1}{2}\dot{h}\dot{\varphi}_2 \\
&+2\cot(\varphi_1)[\dot{\varphi}_1(\delta\varphi_2) + \dot{\varphi}_2(\delta\dot{\varphi}_1)] - 2\csc^2(\varphi_1)\dot{\varphi}_1\dot{\varphi}_2(\delta\varphi_1) \\
&+\frac{2}{R}\dot{\varphi}_2(\delta\dot{R}) - \frac{2}{R^2}\dot{R}\dot{\varphi}_2(\delta R) = 0
\end{aligned}
\tag{0.15}
$$

$$
\begin{aligned}
\ddot{h} \;+\; 2H\dot{h} &= 2\dot{R}(\delta\dot{R}) + 2R^2\dot{\varphi}_1(\delta\dot{\varphi}_1) + 2R^2\sin^2(\varphi_1)\dot{\varphi}_2(\delta\dot{\varphi}_2) \\
&+\; 2\frac{\Omega^2}{a^6R^3}(\delta R) + R^2\sin(2\varphi_1)\dot{\varphi_2}^2(\delta\varphi_1) - V^{'}(R)(\delta R)
\end{aligned}
\tag{0.16}
$$

$$
\begin{aligned}
\dot{h}_{,i} - \dot{h}_{ij,j} = \quad & \dot{R}\partial_i(\delta R) + R^2\dot{\varphi}_1\partial_i(\delta\varphi_1) \\
&+\; R^2\sin^2(\varphi_1)\dot{\varphi}_2\partial_i(\delta\varphi_2)
\end{aligned}
\tag{0.17}
$$

$$
\begin{aligned}
&\frac{1}{a^2}(h_{ij,kk} + h_{,ij} - h_{ik,jk} - h_{jk,ik}) - 3H\dot{h_{ij}} \\
&\qquad -H\dot{h}\delta_{ij} - \ddot{h_{ij}} = \delta_{ij}V^{'}(R)(\delta R)
\end{aligned}
\tag{0.18}
$$

where δR, $\delta\varphi_1$ and $\delta\varphi_2$ are fluctuations of the " radial" and "angular" components respectively. The above equations of motion for fluctuations are the most general case for

quintessence with O(3) internal symmetry. When we consider only the "radial" perturbation, we set φ_1, φ_2, $\delta\varphi_1$ and $\delta\varphi_1$ to zero. If we deal with the perturbation on the "angular" component in the abelian case, what we need to do is to set another "angular" component φ_2 together with its fluctuation $\delta\varphi_2$ to zero. In the subsequent sections, we will discuss them respectively.

4. Perturbation on the "Radial" Components

As we have pointed out in the introduction, the property of the perturbation on the "radial" component depend only on the potential of the model and is independent of whether the internal symmetry is abelian or non-abelian. In this section, we will show this in detail. As we all know that scalar fields with an internal symmetry are likely to produce Q balls or other non-topological solitons, which have been studied by many authors[16, 17, 18]. In their work, they generally put the gravitational effects aside because the magnitude of gravitational fluctuations induced by the fluctuations of the scalar fields are far smaller than the self-interaction of the scalar fields. Firstly, we, following the previous study in this field[16, 17, 18], also investigate the perturbation without considering the metric fluctuation. After this, we take the gravitational effects(metric fluctuations)into account and carry out a similar study on the fluctuations. The equations of motion for fluctuations of the "radial" component are

$$\begin{aligned}(\ddot{\delta R}) + 3H(\dot{\delta R}) - \frac{1}{a^2}\nabla^2(\delta R) + \frac{\Omega^2}{a^6R^4}(\delta R) \\ +V''(R)(\delta R) = 0\end{aligned} \tag{0.19}$$

which is obtained by setting $\delta\varphi_1$, $\delta\varphi_2$ and h in Eq.(0.13) to zero. If we choose for the fluctuation the following form:

$$\delta R(t, \mathbf{x}) = \delta R_0 \exp[\alpha(t) + i\mathbf{kx}] \tag{0.20}$$

then for nontrivial δR_0 , we have

$$\ddot{\alpha} + \dot{\alpha}^2 + 3H\dot{\alpha} - \frac{\Omega^2}{a^6R^4} + \frac{k^2}{a^2} + \frac{\partial^2 V}{\partial R^2} = 0 \tag{0.21}$$

Following the authors in Ref.[16, 17], we assume that $\alpha(t)$ is a slow-varying function, i.e. $\ddot{\alpha}(t) << \dot{\alpha}^2$ and $\dot{\alpha} \approx$ Const. In the following sections, we shall always hold this assumption. Therefore, neglecting the $\ddot{\alpha}$ in the above equation, we have

$$\dot{\alpha} = \frac{1}{2}\Big[-3H \pm \sqrt{(3H)^2 - 4(\frac{k^2}{a^2} - \frac{\Omega^2}{a^6R^4} + \frac{\partial^2 V}{\partial R^2})}\Big] \tag{0.22}$$

If $\dot{\alpha}$ is real and positive, the fluctuation will grow rapidly with time. Therefore the instability band for this fluctuation is

$$0 < k^2 < \frac{\Omega^2}{a^4R^4} - \frac{a^2\partial^2 V}{\partial R^2} \tag{0.23}$$

From Eq.(0.23), one can find that the instability band depends on the specific potential of the quintessence.

When considering the metric fluctuations, one can obtain the following equations of motion for the fluctuations

$$\begin{aligned}(\ddot{\delta R}) + 3H(\dot{\delta R}) - \frac{1}{a^2}\nabla^2(\delta R) + \frac{\Omega^2}{a^6R^4}(\delta R) \\ +V^{''}(R)(\delta R) - \frac{1}{2}\dot{h}\dot{R} = 0\end{aligned} \tag{0.24}$$

$$\ddot{h} + 2H\dot{h} = 2\dot{R}(\dot{\delta R}) + 2\frac{\Omega^2}{a^6R^3}(\delta R) - V^{'}(R)(\delta R) \tag{0.25}$$

$$\dot{h}_{,i} - \dot{h}_{ij,j} = \dot{R}\partial_i(\delta R) \tag{0.26}$$

$$\begin{aligned}\frac{1}{a^2}(h_{ij,kk} + h_{,ij} - h_{ik,jk} - h_{jk,ik}) - 3H\dot{h_{ij}} \\ -H\dot{h}\delta_{ij} - \ddot{h_{ij}} = \delta_{ij}V^{'}(R)(\delta R)\end{aligned} \tag{0.27}$$

If we choose $\Omega = 0$, i.e. in the case of $N = 1$ the Eqs.(0.24)-(0.27) will reduce to Ratra-Peebles's results in the absence of baryonic term[4]. Clearly, since the equations of motion for the metric and scalar fluctuations(Eq.(0.25) and Eq.(0.24)) are linear equations, the fluctuations could be taken as the following form

$$\delta R(t, \mathbf{x}) = \delta R_0 \exp[\alpha(t) + i\mathbf{k}\mathbf{x}] \tag{0.28}$$

$$h(t,\mathbf{x}) = h_0 \exp[\alpha(t) + i\mathbf{k}\mathbf{x}] \tag{0.29}$$

Since there are no $\nabla^2 h$ term in Eq.(0.25), h can not oscillate rapidly and in fact, h will not be able to react, in lowest order, to the rapidly oscillating source terms in Eq.(0.25)[4]. Then for nontrivial δR_0 and h_0, we have

$$\dot{\alpha}^2 + \left(3H - \frac{1}{2}\frac{h_0}{\delta R_0}\dot{R}\right)\dot{\alpha} - \frac{\Omega^2}{a^6 R^4} + \frac{k^2}{a^2} + \frac{\partial^2 V}{\partial R^2} = 0 \tag{0.30}$$

So

$$\begin{aligned} \dot{\alpha} \quad &= \frac{1}{2}\Bigg[- (3H - \frac{1}{2}\frac{h_0}{\delta R_0}\dot{R}) \\ &\pm \sqrt{(3H - \frac{1}{2}\frac{h_0}{\delta R_0}\dot{R})^2 - 4(\frac{k^2}{a^2} - \frac{\Omega^2}{a^6 R^4} + \frac{\partial^2 V}{\partial R^2})}\Bigg] \end{aligned} \tag{0.31}$$

From Eq.(0.31), it is clear that the instability band is the same as that when we did not consider the metric fluctuations. This is reasonable in physics because the metric fluctuations induced by the fluctuations of the field are far smaller than the fluctuations of the field and its back-reaction to the field would be even smaller and thus negligible. So, when considering the metric fluctuations, there should not be a substantial change on the properties of the fluctuation of the quintessence field.

5. Perturbation on the "Angular" Components in the Abelian Case

In this section, we investigate the perturbation on "angular" component in the abelian case, that is, the case in which the quintessence fields possess a O(2) internal symmetry.

By setting δR, h, $\delta\varphi_2$ and φ_2 in Eq.(0.14) to zero, we can obtain the equation of motion for the fluctuation of "angular" component up to the first order as:

$$\delta\ddot{\varphi}_1 + (3H + 2\frac{\dot{R}}{R})\delta\dot{\varphi}_1 - \frac{1}{a^2}\nabla^2\delta\varphi_1 = 0 \tag{0.32}$$

where $\delta\varphi_1$ is the fluctuation of the "angular" component. If we choose

$$\delta\varphi_1(t,\mathbf{x}) = \delta\phi_{10} \exp[\alpha(t) + i\mathbf{k}\mathbf{x}] \tag{0.33}$$

then for nontrivial $\delta\varphi_{10}$, from Eq.(0.32) we have

$$\dot{\alpha}^2 + (3H + 2\frac{\dot{R}}{R})\dot{\alpha} + \frac{k^2}{a^2} = 0 \tag{0.34}$$

It is clear that

$$\dot{\alpha} = \frac{1}{2}\Big[-(3H+2\frac{\dot{R}}{R}) \pm \sqrt{(3H+2\frac{\dot{R}}{R})^2 - 4\frac{k^2}{a^2}}\Big] \tag{0.35}$$

From Eq.(0.35), $\dot{\alpha}$ is always negative and thus the fluctuation will damp quickly. That is, the angular perturbation will not produce a significant "angular" inhomogeneity and the global symmetry will not likely to become space-dependent.

In the following, we will introduce the metric fluctuations into our analysis. By a similar procedure as that in last section, we can obtain the equations of motion for the fluctuations of metric and "angular" component up to the first order as following:

$$\frac{1}{2}\ddot{h} + H\dot{h} = R^2\dot{\varphi}_1\delta\dot{\varphi}_1 \tag{0.36}$$

$$\dot{h}_{,i} - \dot{h}_{ij,j} = R^2\dot{\varphi}_1\partial_i(\delta\varphi_1) \tag{0.37}$$

$$\begin{aligned}\frac{1}{a^2}(h_{ij,kk} + h_{,ij} - h_{ik,jk} - h_{jk,ik}) - 3H\dot{h}_{ij} \\ -H\dot{h}\delta_{ij} - \ddot{h}_{ij} = 0\end{aligned} \tag{0.38}$$

$$\delta\ddot{\varphi}_1 + (3H+2\frac{\dot{R}}{R})\delta\dot{\varphi}_1 - \frac{1}{a^2}\nabla^2\delta\varphi_1 - \frac{1}{2}\dot{h}\dot{\varphi}_1 = 0 \tag{0.39}$$

Since Eq.(0.36)and Eq.(0.39) are linear equations, the fluctuations could be chosen in the following form

$$\delta\varphi_1(t,\mathbf{x}) = \delta\varphi_{10}\exp[\alpha(t) + i\mathbf{k}\mathbf{x}] \tag{0.40}$$

$$h(t,\mathbf{x}) = h_0\exp[\alpha(t) + i\mathbf{k}\mathbf{x}] \tag{0.41}$$

Then for nontrivial $\delta\varphi_{10}$, we have (from Eq.(0.39))

$$\dot{\alpha}^2 + (3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1)\dot{\alpha} + \frac{k^2}{a^2} = 0 \tag{0.42}$$

Therefore, one can obtain

$$\dot{\alpha} = \frac{1}{2}\Big[-(3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1) \pm\sqrt{(3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1)^2 - 4\frac{k^2}{a^2}}\Big] \tag{0.43}$$

From Eq.(0.43), it is clear that $\dot{\alpha}$ will always be negative even if the metric fluctuations are considered.

6. Perturbation on the "Angular" Components in the Non-abelian Case

In this section, we generalize the discussions in last section to the case in which the internal symmetry is non-abelian, that is the symmetry group is O(3). We restrict ourselves to the case that only one "angular" component is perturbed. This will not lose its generality but greatly facilitate the discussion because we can always choose a coordinate system in which the perturbation appears in one angular direction. By setting δR, $\delta\varphi_2$ and h in Eq.(0.14) to zero, we can obtain the equation of motion for the angular fluctuation up to the first order as following:

$$\delta\ddot{\varphi}_1 + (3H + 2\frac{\dot{R}}{R})\delta\dot{\varphi}_1 - \frac{1}{a^2}\nabla^2\delta\varphi_1 - \cos 2\varphi_1 \dot{\varphi}_2{}^2\delta\varphi_1 = 0 \tag{0.44}$$

where φ_1 and φ_2 are the homogeneous parts of the "angular" components. If we choose for $\delta\varphi_1$ the same form as in Eq.(0.40), then for nontrivial $\delta\varphi_{10}$, from Eq.(0.44)we have

$$\dot{\alpha}^2 + (3H + 2\frac{\dot{R}}{R})\dot{\alpha} + \frac{k^2}{a^2} - \cos 2\varphi_1\dot{\varphi}_2{}^2 = 0 \tag{0.45}$$

It is clear that

$$\dot{\alpha} = \frac{1}{2}\Big[-(3H + 2\frac{\dot{R}}{R}) \pm \sqrt{(3H + 2\frac{\dot{R}}{R})^2 - 4\Big(\frac{k^2}{a^2} - \cos 2\varphi_1\dot{\varphi}_2{}^2\Big)}\Big] \tag{0.46}$$

From Eq.(0.46), one can find that $\dot{\alpha}$ could be positive if

$$\frac{k^2}{a^2} - \cos 2\varphi_1\dot{\varphi}_2{}^2 < 0 \tag{0.47}$$

That is, under the above condition(Eq.(0.47)) the "angular" inhomogeneity might grow rapidly with time and make the symmetry group become space-dependent.

When taking into account the metric fluctuation and following a similar process as that in section 5, we can obtain the equations of motion for the fluctuations of "angular" component as follow

$$\begin{aligned}\delta\ddot{\varphi}_1 + (3H \ + \ 2\frac{\dot{R}}{R})\delta\dot{\varphi}_1 - \frac{1}{a^2}\nabla^2\delta\varphi_1 \\ - \ \cos 2\varphi_1 \dot{\varphi_2}^2 \delta\varphi_1 - \frac{1}{2}\dot{h}\dot{\varphi}_1 = 0\end{aligned} \tag{0.48}$$

The equations of motion for metric fluctuations are

$$\ddot{h} \ + \ 2H\dot{h} = 2R^2\dot{\varphi}_1(\delta\dot{\varphi}_1) + R^2\sin(2\varphi_1)\dot{\varphi_2}^2(\delta\varphi_1) \tag{0.49}$$

$$\dot{h}_{,i} - \dot{h}_{ij,j} = R^2\dot{\varphi}_1\partial_i(\delta\varphi_1) \tag{0.50}$$

$$\begin{aligned}\frac{1}{a^2}(h_{ij,kk} + h_{,ij} - h_{ik,jk} - h_{jk,ik}) - 3H\dot{h}_{ij} \\ - H\dot{h}\delta_{ij} - \ddot{h}_{ij} = 0\end{aligned} \tag{0.51}$$

Similarly, we choose the fluctuations to be the form of Eq.(0.40) and Eq.(0.41) and for nontrivial $\delta\varphi_{10}$, from Eq.(0.48) we have

$$\dot{\alpha}^2 + (3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1)\dot{\alpha} - \cos 2\varphi_1\dot{\varphi_2}^2 + \frac{k^2}{a^2} = 0 \tag{0.52}$$

and therefore

$$\begin{aligned}\dot{\alpha} = \frac{1}{2}\Bigg[- (3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1) \pm \\ \sqrt{(3H + 2\frac{\dot{R}}{R} - \frac{1}{2}\frac{h_0}{\delta\varphi_{10}}\dot{\varphi}_1)^2 - 4\left(\frac{k^2}{a^2} - \cos 2\varphi_1\dot{\varphi_2}^2\right)}\Bigg]\end{aligned} \tag{0.53}$$

From Eq.(0.53), one can easily identify that the condition for positive $\dot{\alpha}$ is the same as that in Eq.(0.47). This shows that in the O(3) case, the instability condition for the "angular" fluctuations won't be changed when considering the metric fluctuation, just as that we have proved in the O(2) case.

7. Conclusion And Discussion

In this paper, we investigate the perturbation on both "radial" and "angular" components of the quintessence fields with an internal abelian and non-abelian symmetry. We find that the fluctuation of the "radial" component depends on the specific potential of the quintessence model. Under certain condition, this fluctuation could grow rapidly and thus make the "radial" component space-dependent. For the "angular" perturbation, we find that the properties of the fluctuations depend on whether the internal symmetry group is abelian or non-abelian. In the abelian case, the fluctuation of the "angular" component will damp rapidly with time and therefore the symmetry group will not become space-dependent. In the case that the internal symmetry is non-abelian, we find that under certain condition, the "angular" inhomogeneities might increase rapidly with time and make the symmetry group space-dependent, or local. Here we briefly interpret the physical meaning of this instability: $\delta\varphi_1$ represents a small internal angular inhomogenity; This inhomogenity will increase rapidly under the condition(Eq.(0.47)). Note that this angular inhomogenity is *not* that of space-time. It is surely interesting to study the quintessence with a "local" internal symmetry, which we will investigate in a preparing work. When taking into account the metric fluctuations induced by the fluctuations of the quintessence field and assuming that the back-reaction of the metric fluctuations on the quintessence field are negligible as shown in Ref.[4], the above conclusion still hold true.

It is worth noting that we choose the non-abelian symmetry group as O(3) in this paper. It is not difficult to generalize the O(3) case to O(N) case and one may find that in the O(N) case, the above conclusion for the non-abelian symmetry group still hold true even if the specific condition under which the inhomogeneity increase will change.

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